2224 – 7207M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics: applications and interpretation

Higher level

Paper 2

16 pages

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Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $\mathrm{BC} = 20\ (\mathrm{m})$ ***A1***

[1 mark]

(b) use of Pythagoras ***(M1)***

$\mathrm{AB} = \sqrt{12^2 + 4^2}$

$= 12.6\ (\mathrm{m})\quad (12.6491\ldots,\ \sqrt{160})$ ***A1***

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $\mathrm{A\hat{B}C}$ (or finding the bearing of B from A) ***(A1)***

e.g. $\tan\left(\mathrm{A\hat{B}C}\right) = \frac{12}{4},\ \cos \mathrm{A\hat{B}C} = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649},\ \cos \mathrm{A\hat{B}C} = \frac{6.3245}{20}$

$\mathrm{A\hat{B}C} = 71.6\ (71.5650\ldots)$ ***(A1)***

Note: Angle $\mathrm{A\hat{B}C}$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$180 + 71.5650\ldots = 252°\ (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 180 plus their 71.6. If radians used, award ***A1A1*** for $1.24904\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle *H*)

correct use of a trig ratio to find H, the angle AB makes with horizontal ***(A1)***

e.g. $\tan \hat{H} = \frac{4}{12},\ \cos \hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$

$\hat{H} = 18.4\ (18.4349...)$ ***(A1)***

Note: Accept $18.5\ (18.5078\ldots)$ from use of 3sf answer from part (b). Bearings should be given in degrees.

$270 - 18.4348... = 252°\ (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 270 minus their 18.4. If radians used, award ***A1A1*** for $0.321750\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

[3 marks]

(d) (i) $-\frac{4}{3}$ $\left(-\frac{16}{12}\right)$ ***A1***

(ii) $(6, 8)$ ***A1A1***

Note: Award ***A1A0*** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ ***(M1)***

equation of perpendicular bisector of AC ***(A1)***

e.g. $(y-8)=\frac{3}{4}(x-6)$ **OR** $y=\frac{3}{4}x+3.5$

EITHER

equation of perpendicular bisector of BC is $y=10$ ***(A1)***

OR

equation of perpendicular bisector of AB is $y=-3x+36$ ***(A1)***

Note: The ***A1*** is for either equation of perpendicular bisector of BC or AB.

point of intersection $\left(8\frac{2}{3}, 10\right)$ **OR** $(8.67, 10)$ $\left[(8.666\ldots, 10)\right]$ ***(M1)A1***

Note: Award ***M1*** for an attempt to equate their perpendicular bisectors
Award the final ***A1*** for the correct coordinate pair – parentheses omitted or not.

[8 marks]

[Total: 14 marks]

2. (a) heights, 0, 4, 1.75, 3 and 3.75 seen ***(A2)***

Note: Award ***A1A0*** if **two** of 1.75, 3 or 3.75 are seen.

attempt to use trapezoidal rule formula for their heights ***(M1)***

$\frac{1}{2}\times 1\times\{0+4+2(1.75+3+3.75)\}$ ***(A1)***

Note: Award ***(M1)(A1)*** for correctly expressing this as 3 trapezoids and a triangle. The "$\times 1$" need not be seen.

$=10.5\ (\text{m}^2)$ ***A1***

[5 marks]

(b) $-\frac{1}{12}x^3+x^2+c$ ***A1A1A1***

[3 marks]

(c) $\int_0^4\left(-\frac{1}{4}x^2+2x\right)\text{d}x+1\times 4+\frac{1}{2}\times 7\times 4$ ***(A1)(M1)(A1)***

Note: Award ***A1*** for correct area of rectangle **OR** triangle, ***M1*** for substituting correct limits into given integral (may be seen in part (b)), and ***A1*** for entire expression correct.

$=10.6666\ldots. + 4 + 14$

$= 28\frac{2}{3}\ (\text{m}^2)\left(\frac{86}{3}\right)$ ***A1***

Note: The answer must be **exact** for the ***A1*** to be awarded. For an answer of 28.7 or 28.66 award ***(A1)(M1)(A1)A0***.

[4 marks]

(d) (Total area using part (a) =) 28.5 ***(A1)***

Percentage error $=\left|\frac{28.5-28.6666\ldots}{28.6666\ldots}\right|\times 100$ ***(M1)***

Note: if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most ***A0M1A0***. If it is clear from the answer that ×100 has been used, then condone the omission and award the ***M*** mark.

$=0.581\ (\%)\ (0.581395\ldots)$ ***A1***

(accept 0.697 from use of 28.7)

[3 marks]

[Total: 15 marks]

3. (a) (i) correct approach to find missing length ***(A1)***

$\sqrt{4^2-1^2}\quad(=\sqrt{15})$

attempt to find cross-section ***(M1)***

e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle

use of volume of prism formula ***(M1)***

(their cross-section multiplied by 3)

$3\left[\frac{1}{2}(10+11)\left(\sqrt{4^2-1^2}\right)\right]$

$=122\,(\text{m}^3)\ \ (121.998\ldots)$ ***A1***

(ii) correct approach to find missing height ***(A1)***

$\sqrt{4^2-3.2^2}\quad(=2.4)$

attempt to find volume ***(M1)***

(multiplication by 3.2 and 3 seen)

$3\left[\frac{1}{2}\left(10+10+\sqrt{4^2-3.2^2}\right)(3.2)\right]$

$=108\,(\text{m}^3)\ \ (107.52...)$ ***A1***

(iii) correct approach to find missing lengths ***(A1)***

$\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ **OR** $\sin\left(\frac{\pi}{3}\right)$ and Pythagoras etc seen in work

$3\left[\frac{1}{2}(10+10+4\cos\left(\frac{\pi}{3}\right))4\sin\left(\frac{\pi}{3}\right)\right]$

$=114\,(\text{m}^3)\ \ (114.315\ldots)$ ***A1***

[9 marks]

(b) $V=3\left[\frac{1}{2}(10+10+4\cos(\theta))4\sin(\theta)\right]$ ***A1***

all correct intermediate working leading to given answer ***A1***

e.g. $V=6\sin(\theta)(20+4\cos(\theta))$

$V=24\sin(\theta)(5+\cos(\theta))$ ***AG***

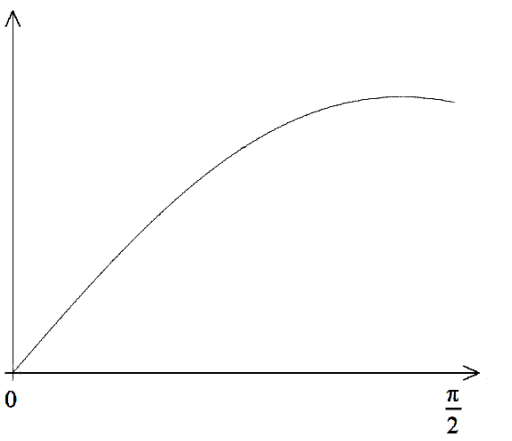
Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(c) *accept any reasoning along the lines:* "skip would have zero volume" or "if the angle is zero, then the contents would fall out" ***R1***

[1 mark]

(d) (i)



A1A1

Note: Award ***A1*** for the correct shape and ***A1*** for the graph on the correct, labelled, domain. Condone omission of θ / V labels (or x/y).

(ii) $\theta = 1.38 \ (1.38356...) \ (79.3^\circ \ (79.2723...^\circ))$ ***A1***

$V_{\max} = 122 \ (122.292...)$ ***A1***

Note: Award ***A0A1*** if values are reversed and ***A0A0*** for a coordinate pair.

[4 marks]

(e) recognizing that derivative is equal to zero (seen at any stage) ***M1***

$\frac{dV}{d\theta} = 0$ (accept $\frac{dy}{dx} = 0$)

(from graph, turning point is a global maximum)

use of product rule ***M1***

$\left(\frac{dV}{d\theta} =\right) 24\cos(\theta)\left(5 + \cos(\theta)\right) + 24\sin(\theta)\left(-\sin(\theta)\right)$ ***A1***

$= 120\cos(\theta) + 24\cos^2(\theta) - 24\sin^2(\theta) \ (= 0)$ (or equivalent) ***A1***

substituting $1 - \cos^2(\theta)$ for $\sin^2(\theta)$ ***M1***

e.g $120\cos(\theta) + 24\cos^2(\theta) - 24(1 - \cos^2(\theta)) \ (= 0)$

correct intermediate steps leading to given answer ***A1***

$2\cos^2(\theta) + 5\cos(\theta) - 1 = 0$ ***AG***

[6 marks]

[Total: 22 marks]

4. (a) AEDCFBA ***A1A1***

Note: Award ***A1*** for AE at start, ***A1*** for correct completed route.

attempt to find the length of their route
length $22+21+19+24+25+31$ ***(M1)***
$=142$ (km) ***A1***

Note: Award ***A1A0M1A0*** for omitted final edge and their sum.

[4 marks]

(b) attempt to form MST without vertex A ***(M1)***

Note: Exactly 4 edges that form a spanning tree are required.

BD DC DE DF **OR** 20, 19, 21, 22 seen in that order ***A1***

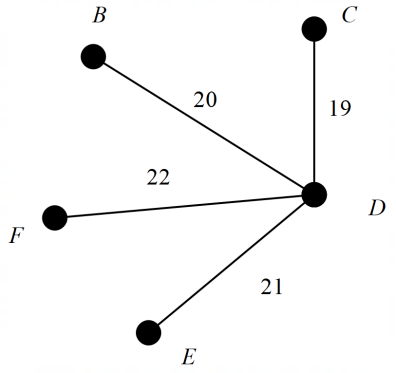
Note: Award ***M1A0*** for diagram of MST.

attempt to reconnect vertex A (one edge is sufficient) ***(M1)***
reconnecting A: AE (22) and AF (23) ***(A1)***
lower bound: $20+19+21+22+22+23$
$=127$ ***A1***

Note: If 127 seen, unsupported or without the explicit evidence of Prim's algorithm, award ***M1A0M1A1A1***.

[5 marks]

(c)



(A1)

Note: Condone the omission of the weights from their diagram. The diagram may include A with its two edges.

correct reasoning based on lack of cycle (once A is reattached) ***R1***
e.g. edges BD and CD would be repeated
this lower bound is not achievable (in this way) ***A1***

Note: Do not award ***R0A1***.

[3 marks]
[Total: 12 marks]

5. (a) $\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}$ ***M1A1***

Note: Award ***M1*** for correct values used, ***A1*** if in correct positions.
Accept alternative consistent matrix (e.g. the transpose or diagonal elements exchanged) and follow through to eigenvectors and initial state vector.

[2 marks]

(b) 5 (seen) ***(A1)***

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}^5 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.596608 \\ 0.403392 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}^5 = \begin{pmatrix} 0.596608 & 0.268928 \\ 0.403392 & 0.731072 \end{pmatrix}$ ***(M1)***

$\mathrm{P}(\text{Friday evening}) = 0.403 \ \ (0.403392)$ ***A1***

Note: Award ***A0M1A0*** for use of 4 (and resulting probability 0.354).

[3 marks]

(c) attempt to find $\det(\boldsymbol{A} - \lambda \boldsymbol{I})$ ***(M1)***

$\begin{vmatrix} 0.88-\lambda & 0.08 \\ 0.12 & 0.92-\lambda \end{vmatrix}$ **OR** $(0.88-\lambda)(0.92-\lambda) - (0.12)(0.08)$

$\lambda^2 - 1.8\lambda + 0.8$ ***A1***

[2 marks]

(d) eigenvalues are 0.8 and 1 ***(A1)***

Note: If no attempt is made to find eigenvectors, do not award ***A1*** for finding eigenvalues.

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0.8 \begin{pmatrix} x \\ y \end{pmatrix}$

$0.88x + 0.08y = 0.8x$

eigenvector = eg. $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1***

EITHER

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.88x + 0.08y = x$

$0.08y = 0.12x$

OR

eigenvalue 1 gives

$\begin{pmatrix} -0.12 & 0.08 \\ 0.12 & -0.08 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

$-0.12x + 0.08y = 0$

$0.08y = 0.12x$

Note: Award ***M1*** for an attempt to find the eigenvector with eigenvalue 1.

THEN

eigenvector = eg. $\begin{pmatrix}2\\3\end{pmatrix}$ ***A1***

Note: Award ***A0A1M0A0*** if only $\begin{pmatrix}1\\-1\end{pmatrix}$ is seen and no eigenvalues are found.

[4 marks]

(e) $\boldsymbol{D}=\begin{pmatrix}1 & 0\\0 & 0.8\end{pmatrix}$, $\boldsymbol{P}=\begin{pmatrix}2 & 1\\3 & -1\end{pmatrix}$ **OR** $\boldsymbol{D}=\begin{pmatrix}0.8 & 0\\0 & 1\end{pmatrix}$, $\boldsymbol{P}=\begin{pmatrix}1 & 2\\-1 & 3\end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for one of $\boldsymbol{P}$ or $\boldsymbol{D}$ correct. Do not award the second ***A1*** unless $\boldsymbol{P}$ and $\boldsymbol{D}$ are consistent.

[2 marks]

(f) **EITHER**

attempt to use $\boldsymbol{T}^n=(\boldsymbol{PDP}^{-1})^n=\boldsymbol{PD}^n\boldsymbol{P}^{-1}$ ***M1***

Note: Award ***M1*** for their $\boldsymbol{D}^n$ seen.

limit of $\boldsymbol{D}^n$ calculated ***A1***

$\begin{pmatrix}2 & 1\\3 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}2 & 1\\3 & -1\end{pmatrix}^{-1}$

Note: $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$ must be seen to award ***A1***.

OR

attempt to expand their $\boldsymbol{PD}^n\boldsymbol{P}^{-1}$ using explicit $\boldsymbol{P}$, $\boldsymbol{P}^{-1}$ ***M1***

$(\boldsymbol{T}^n=)\ \frac{1}{5}\begin{pmatrix}2 & 1\\3 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0.8^n\end{pmatrix}\begin{pmatrix}1 & 1\\3 & -2\end{pmatrix}$

$(\boldsymbol{T}^n=)\ \frac{1}{5}\begin{pmatrix}2+3(0.8^n) & 2-2(0.8^n)\\3-3(0.8^n) & 3+2(0.8^n)\end{pmatrix}$ ***A1***

Note: Using this method, the limit of 0.8^n may be inferred and ***M1A1*** awarded.

THEN

0.6 ***A1***

Note: Multiplication by initial condition $\begin{pmatrix}1\\0\end{pmatrix}$ may be seen at any point as part of their method.

For an answer of 0.6 from incomplete methods award a maximum of ***M1A0A0***, or if no working is seen, award ***M0A0A1***.

[3 marks]

[Total: 16 marks]

6. (a) (i) 15 ***A1***

(ii) **EITHER**
attempt to use arithmetic series formula ***(M1)***
OR
attempt to set up simultaneous equations ***(M1)***
OR
attempt to use quadratic regression ***(M1)***
$(T_k =)\frac{1}{2}k^2 + \frac{1}{2}k$ ***A1A1***

Note: Condone variable change (eg in quadratic regression).
Accept $a = \frac{1}{2}$, $b = \frac{1}{2}$.

[4 marks]

(b) (i) $(15 + 10 =)\ 25$ ***A1***

(ii) $\frac{k(k+1)}{2} + \frac{(k-1)((k-1)+1)}{2}$ **OR** $\frac{1}{2}k^2 + \frac{1}{2}k + \frac{1}{2}(k-1)^2 + \frac{1}{2}(k-1)$ ***(A1)***
$= k^2$ ***A1***

[3 marks]

(c) one correct product of probabilities seen: $\frac{15}{25} \times \frac{10}{24}$ **OR** $\frac{10}{25} \times \frac{15}{24}$ ***(A1)***
adding their products ***(M1)***
$\frac{15}{25} \times \frac{10}{24} + \frac{10}{25} \times \frac{15}{24}$
$= \frac{1}{2}$ ***A1***

[3 marks]

(d) attempt to add two products of probabilities involving k only ***M1***
(these may be incorrect or in terms of T_k)
$\frac{\frac{k}{2}(k+1)}{k^2} \times \frac{\frac{k}{2}(k-1)}{k^2-1} + \frac{\frac{k}{2}(k-1)}{k^2} \times \frac{\frac{k}{2}(k+1)}{k^2-1}$ ***A1***

further simplification consistent with given answer ***A1***
$= \frac{1}{2}$ ***A1***
hence independent of k ***AG***

[4 marks]
[Total: 14 marks]

7. (a) $\mathrm{P}(X < 340)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 0.159 \quad (0.158656\ldots)$ ***A1***

[2 marks]

(b) recognizing endpoint occurs at either 0.975 or 0.025 ***(M1)***

$\mathrm{P}(X < k) = 0.975$ **OR** $\mathrm{P}(X < m) = 0.025$

$330 < X < 370 \quad (330.400... < X < 369.599...)$ ***A1A1***

[3 marks]

(c) (i) recognizing mean of W is sum of individual means within wall ***(M1)***

$W = C_1 + C_2 + L$ may be seen

$\mathrm{E}(W) = 2\mathrm{E}(C) + \mathrm{E}(L)$

$= 800$ ***A1***

recognizing **variance** of W is sum of individual **variances** within wall ***(M1)***

$\mathrm{Var}(W) = 2\mathrm{Var}(C) + \mathrm{Var}(L)$ **OR** 225 seen ***(A1)***

$\left(\mathrm{SD}(W) =\right) \ 15$ ***A1***

Note: Award ***M1A0A0*** for an answer of 20.6 from using $\mathrm{Var}(2C)$ in place of $2\mathrm{Var}(C)$.

(ii) recognizing that W is modelled by a normal distribution ***(M1)***

$\left(\mathrm{P}(780 < W < 810) =\right) \ 0.656 \quad (0.656296...)$ ***A1***

Note: The answer is $0.521 \ (0.520)$ from using $\mathrm{SD} = 20.6 \quad (5\sqrt{17})$. Follow through from part (c)(i) without working seen.

[7 marks]

(d) $810 = 350 + 350 + \mathrm{E}(L)$ (or equivalent) ***(A1)***

$\left(\mathrm{E}(L) =\right) \ 110$ ***A1***

$\mathrm{Var}(W) = 2\mathrm{Var}(C) + \mathrm{Var}(L)$ **OR** $256 = 2(100) + \mathrm{Var}(L)$ ***(A1)***

$\left(\mathrm{SD}(L) =\right) \ 7.48 \ (7.48331..., \sqrt{56})$ ***A1***

[4 marks]

(e) $116 \quad (116.298)$ ***A1***

Note: Do not follow through from either a negative variance or a negative SD.

[1 mark]

[Total: 17 marks]